ON EXCEPTIONAL VALUES OF A MEROMORPHIC FUNCTION

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1. M. Brelot [1] has shown that if $u(z)$ is subharmonic in an open set D in the z-plane with boundary C and is bounded from above in a neighborhood of a boundary point z_0, which is contained in a set $E \subset C$ of inner harmonic measure zero with respect to D, and such that z_0 is a regular point for Dirichlet problem in D, then

$$(1) \qquad \overline{\lim_{\substack{z \to z_0 \\ z \in D}}}\, u(z) = \overline{\lim_{\substack{z' \to z_0 \\ z' \in C-E}}}\, \Big(\overline{\lim_{\substack{z \to z' \\ z \in D}}}\, u(z)\Big).$$

Furthermore, it was shown that if $f(z)$ is meromorphic in D, then, for any z_0 of E, which is in the closure of $C-E$, whether a regular point or not, the same relation holds when $u(z)$ is replaced by $|f(z)|$ whenever the left side of (1) is finite. It is easy to see that this last relation is equivalent to the relation:[1]

$$(2) \qquad \textit{boundary of } S_{z_0}^{(D)} \subset S_{z_0}^{(C-E)},$$

where the cluster set $S_{z_0}^{(D)}$ is the set of values approached sequencewise by $f(z)$ in any neighborhood of z_0 and the boundary cluster set $S_{z_0}^{(C-E)}$ from $C-E$ is the limit of the closure of $\bigcup_{z' \in (C-E)_r} S_{z'}^{(D)}$ as $r \to 0$, $(C-E)_r$ being that part of $C-E$ in $|z-z_0| < r$.

Later M. Tsuji [5] showed that in the special case that D is a domain and E is a closed set of logarithmic capacity zero, the exceptional values in $\Omega = S_{z_0}^{(D)} - S_{z_0}^{(C-E)}$, that is, the set of values in Ω which $f(z)$ does not assume in some neighborhood of z_0 form a set of inner logarithmic capacity zero.

2. In this note we shall prove that this is true in the general case.

THEOREM. *Let D be an open set in the z-plane, C its boundary, $E \subset C$ a set of inner harmonic measure zero with respect to D, z_0 a point of E in the closure of $C-E$, and $f(z)$ a meromorphic function in D. Then every value of*

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[1] See [4], for instance.