AN EXAMPLE OF NORMAL LOCAL RING WHICH IS ANALYTICALLY RAMIFIED

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Previously the following question was offered by Zariski [6]:

Is any normal Noetherian local ring analytically irreducible?[1)]

In the present note, we will construct a counter-example against the question.

TERMINOLOGY. A ring (integrity domain) means always a commutative ring (integrity domain) with identity. A normal ring is an integrity domain which is integrally closed in its field of quotients. When $\mathfrak{o}$ is an integrity domain, the integral closure of $\mathfrak{o}$ in its field of quotients is called the derived normal ring of $\mathfrak{o}$.

In our treatment, some basic notions and results on general commutative rings and Noetherian local rings are assumed to be well known (see, for example, [5] and one of [1] or [2]). In particular, some results on regular local rings and completions of local rings are used freely (without references). On the other hand, we will make use of an example constructed in [3, § 1] without proof.

§ 1. The construction of an example

Let $\mathbf{k}_0$ be a perfect field of characteristic 2 and let $u_0, v_0, \ldots, u_n, v_n, \ldots$ (infinitely many) be algebraically independent elements over $\mathbf{k}_0$. Set $\mathbf{k} = \mathbf{k}_0(u_0, v_0, \ldots, u_n, v_n, \ldots)$. Further let x and y be indeterminates and set $\mathfrak{r} = \mathbf{k}\{x, y\}$ (formal power series ring), $\mathfrak{o} = \mathbf{k}^2\{x, y\}[\mathbf{k}]$ and $c = \sum_{i=0}^{\infty}(u_i x^i + v_i y^i)$. Then we set $\mathfrak{S} = \mathfrak{o}[c]$.

PROPOSITION. *$\mathfrak{S}$ is a normal Noetherian local ring and the completion of $\mathfrak{S}$ contains non-zero nilpotent elements (that is, $\mathfrak{S}$ is analytically ramified).*

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1) It was conjectured that the answer is negative by [4] and the present paper answers the conjecture affirmatively.