ON AFFINE TRANSFORMATIONS OF A RIEMANNIAN MANIFOLD*

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In this paper we establish some theorems about the group of affine transformations on a Riemannian manifold. First we prove a decomposition theorem (Theorem 1) of the largest connected group of affine transformations on a simply connected complete Riemannian manifold, which corresponds to the decomposition theorem of de Rham [4][1] for the manifold. In the case of the largest group of isometries, a theorem of the same type is found in de Rham's paper [4] in a weaker form. Using Theorem 1 we obtain a sufficient condition for an infinitesimal affine transformation to be a Killing vector field (Theorem 2). This result includes K. Yano's theorem [13] which states that on a compact Riemannian manifold an infinitesimal affine transformation is always a Killing vector field. His proof of the theorem depends on an integral formula which is valid only for a compact manifold. Our method is quite different and is based on a result [11] of K. Nomizu.

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I. Preliminaries

1. Let M be a differentiable manifold of class C^∞.[2] The set $\mathfrak{X}$ of all tangent vector fields defined on M is a module over the ring $\mathfrak{F}$ of all differentiable functions on M.

An affine connection is defined by a homomorphism over $\mathfrak{F}$: $X \to \nabla_X$ from $\mathfrak{X}$ into the module of linear mappings (over the field of all real numbers) of T, which satisfies the following condition

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1) Numbers in brackets refer to Bibliography at the end of this paper.

2) As we only consider manifolds, tangent vector fields, tensor fields and mappings which are "differentiable of class C^∞," we always omit this adjective. We deal only with connected manifolds. For the terminology concering manifolds, we follow C. Chevalley [3].