ON A PROBLEM OF CHEVALLEY

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Recently Prof. Chevalley in Nagoya suggested to the author the following problem: Let k be a field, $K_5 = k(x_1, x_2, x_3, x_4, x_5)$ be a purely transcendental extension field (of transcendental degree 5) of k, s_5 be the cyclic permutation of x: $s_5x_1 = x_2 s_5x_2 = x_3 s_5 x_3 = x_4 s_5 x_4 = x_5 s_5 x_5 = x_1$, and let L_5 be the field of invariants of s_5 in K_5. Is L_5 then purely transcendental over k or not? When the characteristic p of k is not equal to 5, it is answered in the following positively. When the characteristic p of k is equal to 5, it is answered also positively by Mr. Kuniyoshi's result in [2].

Now let $K_n = k(x_1, x_2, x_3, \ldots, x_n)$ be a purely transcendental extension field (of transcendental degree n) of k, s_n be the cyclic permutation of x: $s_nx_1 = x_2$ $s_nx_2 = x_3 \ldots s_nx_n = x_1$, and let L_n be the field of invariants of s_n in K_n. We suppose from now on throghout the present article that n is not divisible by the characteristic p of k. If the ground field k involves a primitive n-th root ζ_n of 1, we can see easily that L_n is purely transcendental over k. From this fact we obtain in the following that existence of certain sets of primitive generators of $L_n(\zeta_n)$ over $k(\zeta_n)$ (the definition is shown in the following) is a necessary and sufficient condition for L_n to be purely transcendental over k, and the existence of such sets of primitive generators are shown for every case of $n \leqq 7$ through calculations on factor sets[1]. It looks that a more arithmetical approach will be necessary to solve the problem with reference to general n.

1. Let $k'_n = k(\zeta_n)$, $K'_n = K_n(\zeta_n)$ and $\mathfrak{G}$ be the Galois group of k' over k. We omit all n as subscripts throughout in the following, unless indispensable. Let L' denote the field of invariants of s in K'. K and K' are clearly Galois extension fields over L and L' of the same rank n respectively. Their Galois groups are generated by the automorphism induced by s. We do not distin-

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[1] Cf. [1] and [3]. Hasse factor sets defined in [3] without the supposition that the absolutely irreducible representations of Galois groups are obtained in the ground field has close relations to the problem of the pure transcendency of L_n over k.