BOUNDARY COMPONENTS OF RIEMANN SURFACES*

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Introduction. The boundary components of an abstract Riemann surface were defined by B. v. Kérékjártó [7] and utilized in the book [14] written by S. Stoïlow.[1] It is the purpose of the present paper to investigate their images under conformal mapping and to solve the Dirichlet problem with boundary values distributed on them.

Suppose that the universal covering surface of a Riemann surface $\mathfrak{R}$ is of hyperbolic type, that is, conformally equivalent to the disc $U: |z|<1$. The work [15] by M. Tsuji shows that the linear measure of the image on $\Gamma: |z|=1$ of the set $\mathfrak{C}_{\mathfrak{R}}$ of all boundary components of $\mathfrak{R}$ is 0 or 2π according as $\mathfrak{R}$ has a null or positive boundary. The writer in [8] studied topologically the image on Γ of each boundary component. In Chapter I of the present paper we shall continue this study.

The set $\mathfrak{C}_{\mathfrak{R}}$ may be regarded as a topological space, as was done by Stoïlow [14]. We are naturally led to consider the Dirichlet problem on $\mathfrak{R}$ with boundary values on $\mathfrak{C}_{\mathfrak{R}}$, with respect to this topology. We shall treat this problem in Chapter II by the Perron-Brelot's method; it was proposed in [8] but left open there.[2]

Chapter I. Boundary Correspondence

1. Definition of boundary components. Throughout this paper let $\mathfrak{R}$ be an

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1) Kérékjártó and Stoïlow called them Randstücke and éléments-frontières respectively. The writer used the term "ideal boundary component" in [8] but now drops the word "ideal."

2) It was pointed out in the lecture given by M. Brelot at the conference at Michigan in 1953 (see [4]) that the solution of this problem follows also from the results in [5].