# ON SOME ASYMPTOTIC PROPERTIES OF POISSON PROCESS

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The Poisson process $X(t, \omega)$,[1] $(\omega \in \Omega,\ 0 \leqq t < \infty)$, as is well-known, is a temporally and spatially homogeneous Markoff process satisfying

(1) $X(0, \omega) = 0$ and $X(t, \omega) =$ integer $\geqq 0$ for every $\omega \in \Omega$,

(2) $$Pr\{X(t, \omega) - X(t', \omega) \geqq k\} = \sum_{i=k}^{\infty} \frac{\{\lambda(t-t')\}^i}{i\,!} e^{-\lambda(t-t')} \quad \text{for} \quad t > t',$$

where $k$ is a non-negative integer and $\lambda$ is a positive constant. In this note we consider the random variable $L_m(\omega)$ which denotes the length of $t$-interval such that $X(t, \omega) = m$ $(m = 0, 1, 2, \ldots)$ and some of other properties concerning them.

**§ 1. The known results on $L_m$.**

*Definition. We define $L_m(\omega)$, the function of $m$ and $\omega$, as follows,*

$$L_m(\omega) = t_{m+1}(\omega) - t_m(\omega),$$

*where*

$$t_m(\omega) = \text{Min}\{\tau;\ X(\tau, \omega) = m\}.$$

This $t_m(\omega)$ exists almost certainly by the right continuity property of Poisson process, and furthermore it is clear that $t_m(\omega)$ is measurable. Thus $L_m(\omega)$ becomes a non-negative random variable.

THEOREM 1. *$L_0, L_1, \ldots, L_m, \ldots$ are mutually independent random variables with a common distribution function $F(l)$,*
*where*

(3) $$F(l) = \begin{cases} 1 - e^{-\lambda l} & \text{if } l \geqq 0, \\ 0 & \textit{otherwise}. \end{cases}$$

*Furthermore*

(4) $$E(L_m)^{2)} = \frac{1}{\lambda}$$

(5) $$V(L_m)^{2)} = \frac{1}{\lambda^2} \qquad m = 0, 1, 2, \ldots$$

This theorem was already suggested by P. Levy [2][3] and a rigorous proof was

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1) $\omega$ denotes the probability parameter.

2) $E(\ldots)$ and $V(\ldots)$ denote the mean and the variance respectively.

3) Numbers in brackets refer to the bibliography at the end of this note.