CORRECTIONS TO MY PAPER "ON THE STRUCTURE OF COMPLETE LOCAL RINGS"[1]

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The proof of Proposition 2 and that of Corollary to Proposition 3 in my previous paper "On the structure of complete local rings"[1] are not correct.[2] Here we want to correct them.

Proof of Proposition 2.

Since the previous proof of Proposition 2 is valid when $R/\mathfrak{m}$ is perfect, we treat only the case when $R/\mathfrak{m}$ is not perfect.

Starting from $K_0 = R/\mathfrak{m}$, we obtain K_n $(n = 1, 2, \ldots)$ from K_{n-1} by adjoining all p-th roots of elements of K_{n-1}.

Definition. Let a local ring R_1 with maximal ideal $\mathfrak{m}_1$ be a subring of another local ring R_2 with maximal ideal $\mathfrak{m}_2$. We say that R_2 is unramified with respect to R_1 if $\mathfrak{m}_2 = \mathfrak{m}_1 R_2$ and $\mathfrak{m}_2^k \cap R_1 = \mathfrak{m}_1^k$ for every positive integer k.

(1) Equal characteristic case.

We construct a sequence of local rings $R = R^{(0)} \subset R^{(1)} \subset \ldots$ such that (1) $R^{(n)}$ is unramified with respect to R, (2) $R^{(n)}/\mathfrak{m}R^{(n)} = K_n$ and (3) $(R^{(n)})^p \subseteq R^{(n-1)}$.

The existence of such a sequence obviously follows from Zorn's Lemma if we observe that a monic polynomial $f(x)$ over a local ring, say R^*, is irreducible modulo its maximal ideal, then $R^*[x]/(f(x))$ is unramified with respect to R^*. (We may use the p-basis).

Let S be the union of all $R^{(n)}$. Then S is a local ring unramified with respect to R. For every element a^* of $R/\mathfrak{m}$, we construct a sequence (a_n) as follows: Let b_n be a representative of $a^{*p^{-n}}$ in R_n and set $a_n = b_n^{p^n}$. Then $a_n \in R$ and the limit a, which is the multiplicative representative of a^*, is in R. Thus we have Proposition 2 in this case.

(2) Unequal characteristic case.

As in above, we construct a sequence of local rings $R = R^{(0)} \subset R^{(1)} \subset \ldots$ satisfying the above conditions (1) and (2) as follows: Let $\mathfrak{M} = \mathfrak{M}^{(0)}$ be a sys-

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[1] Nagoya Math. Journ. 1 (1950), pp. 63-70.

[2] Prof. I. S. Cohen (Massachusetts Institute of Technology, U.S.A.) pointed out the error of the proof of Proposition 2. I am grateful to him for his kind communication.