UNITARY REPRESENTATIONS OF SOME LINEAR GROUPS II

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§ 0. Introudction. In his preceding paper [2], the author determined the types of irreducible unitary representations and cyclic unitary representations of the group of all euclidean motions in 2-space E^2. The purpose of the present paper is to determine the types of irreducible unitary representations and cyclic ones of the group of all euclidean motions in n-space E^n for $n \geqq 3$.[1),2)] In this paper, we shall make use of the results of the preceding paper [2], but notations are independent of those in [2].

§ 1. Preliminaries and main theorems. Let $\mathbf{G}$ be the group of all euclidean motions in n-space E^n. Then $\mathbf{G}$ has a compact subgroup $\mathbf{K} \cong SO(n)$ and a normal subgroup $\mathbf{V}$ isomorphic to the vector group R^n, and

$$(1.1) \qquad \begin{cases} \mathbf{G} = \mathbf{V} \cdot \mathbf{K}, \quad \mathbf{V} \cap \mathbf{K} = \{e\} \quad (e = \text{the identity of } \mathbf{G}) \\ \mathbf{G}/\mathbf{V} \cong \mathbf{K}. \end{cases}$$

Let X be the character group of $\mathbf{V}$, and χ_0 be the identity of X; then $X \cong R^n$. Hereafter $g, g', \ldots$ denote elements of $\mathbf{G}$, especially $a, b, c, \ldots$ —— of $\mathbf{K}$, $x, y, \ldots$ —— of $\mathbf{V}$; and $\chi, \chi', \ldots$ —— elements of X. (χ, x) denotes the value of character χ at $x \in \mathbf{V}$. We denote by M_a the orthogonal matrix which realize the element $a \in \mathbf{K}$ and by M_a^* its conjugate matrix, and define that $M_a x$ means to operate M_a to x as a vector in R^n while ax and xa mean the multiplications as elements of the group $\mathbf{G}$. We shall denote briefly χa instead of $M_a^* \chi$. Then, if

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, \quad \chi = (\chi_1, \ldots, \chi_n) \text{ and } M_a = \begin{pmatrix} a_{11} & \ldots & a_{1n} \\ \ldots & \ldots & \ldots \\ a_{n1} & \ldots & a_{nn} \end{pmatrix},$$

Received August 12, 1952.

1) The author wrote in [2] that it seemed to be difficult to solve such problem for $n \geqq 3$. But he could solve this problem after he finished the proof-reading of the paper [2].

2) Prof. G. W. Mackey kindly informed to the author that the result of [2] was inculuded in the result of his paper [3] which the author had overlooked. Recently more general cases have been treated in [4] and [5]. However, the results of the papers [3], [4] and [5] seem to be not so explicit as the result of our present paper.