ON A THEOREM OF H. F. BLICHFELDT

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In 1903 H. F. Blichfeldt[1] proved the following brilliant theorem: Let G be a matrix group of order g and of degree n. Let p be a prime divisor of g such that $p>(n-1)(2n+1)$. Then G contains the abelian normal p-Sylow subgroup. In 1941 applying his modular theory of the group representation, R. Brauer[2] improved this theorem in the case in which p divides g to the first power only. Further in 1943 H. F. Tuan[3] improved this result of R. Brauer one step more.

Now in the present paper we prove the following

THEOREM. *Let G be a Soluble matrix group of order g and of degree n. Let p be a prime divisor of g such that $p>n$. Then either* (i) *G contains the normal abelian p-Sylow subgroup or* (ii) *p is a Fermat prime $p=n+1$, $n=2^m$, g is even and G contains a subgroup of peculiar type* (*See the proof below*). (*The coefficient field is the field of all complex numbers.*)

Proof. Let P be a p-Sylow subgroup of G. Let P be not abelian. Then the degree of a faithful representation of P is greater than $p(>n)$. Hence P must be abelian. Therefore we have only to prove that P is normal in G, provided that the case (ii) does not occur.

Now we prove this by induction arguments with respect to the degree and the order of the group. In particular we assume that the p-Sylow subgroup is normal in all the proper subgroups of the group G.

Let G be reducible. Then G is decomposable and we may assume that $G=\begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix}$. Let $P=\begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix}$ be a p-Sylow subgroup of G. Since G_i contains the normal p-Sylow subgroup by induction hypothesis, P_i is the normal p-Sylow subgroup of G_i ($i=1, 2$). Then P is normal in G. Hence we may assume that G is irreducible.

Received June 19, 1952.

1) On the order of linear homogeneous groups, Transactions Am. Math. Soc., vol. 4 (1903), 387-397.

2) On groups whose order contains a prime number to the first power II, American Journal of Mathematics, vol. 54 (1942), 421-440.

3) On groups whose orders contain a prime number to the first power, Annals of Mathematics, vol. 45 (1944), 110-140.