NOTE ON A-GROUPS

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Let us consider soluble groups whose Sylow subgroups are all abelian. Such groups we call A-groups, following P. Hall. A-groups were investigated thoroughly by P. Hall and D. R. Taunt from the view point of the structure theory.[1] In this note, we want to give some remarks concerning representation theoretical properties of A-groups.

§ **1. Definition.** A group $\mathfrak{G}$ is called an M-group if all its irreducible representations are similar to those of monomial forms.

PROPOSITION 1. Every A-group is an M-group.

Proof. Let $\mathfrak{G}$ be an A-group and let $\mathfrak{Z}$ be an irreducible representation of $\mathfrak{G}$. Obviously the A-property is hereditary to subgroups and factor groups. Therefore, applying the induction argument with respect to the order of $\mathfrak{G}$, we see that we have only to consider faithful, primitive irreducible representations of $\mathfrak{G}$. Let $\mathfrak{Z}=\mathfrak{G}$ be such a one. Let $\mathfrak{N}$ be the radical, that is, the largest nilpotent normal subgroup of $\mathfrak{G}$. Since $\mathfrak{G}$ is an A-group, the radical $\mathfrak{N}$ is abelian. Therefore by a theorem of H. Blichfeld,[2] $\mathfrak{N}$ must coincide with the centre of $\mathfrak{G}$. If $\mathfrak{G}=\mathfrak{N}$, the assertion is trivial. If $\mathfrak{G}\neq\mathfrak{N}$, let $\mathfrak{N}_1$ be a normal subgroup of $\mathfrak{G}$, which is minimal over $\mathfrak{N}$. Then obviously $\mathfrak{N}_1$ is nilpotent and therefore $\mathfrak{N}_1=\mathfrak{N}$ which is a contradiction. Q.E.D.

Imposing some strong restriction on $\mathfrak{G}$, M. Tazawa proved the proposition 1.[3]

The M-property is not always hereditary to subgroups. First we remark the following well known fact:

(A) Let us consider a matrix group $\mathfrak{M}$ whose character is denoted by χ. Then $\mathfrak{M}$ is irreducible if and only if $\sum\chi\bar{\chi}=$ the order of $\mathfrak{M}$.

Example. Let $\mathfrak{G}$ be the hyperoctahedral group of degree 4 (and of order $2^4 \cdot 4!$). Then $\mathfrak{G}$ is irreducible, which is easily verified applying (A). Let

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[1] P. Hall, The construction of soluble groups. J. Reine Angew. Math. **182**, 206-214 (1940). D. R. Taunt, On A-groups. Proc. Cambridge Philos. Soc. **45**, 24-42 (1949). The latter is not yet accessible to me.

[2] H. Blichfeld, Finite Collineation Groups. Chicago (1917).

[3] M. Tazawa, Über die monomial darstellbaren endlichen Substitutionsgruppen. Proc. Acad. Jap. **10**, 397-398 (1934).