ON THE DIFFERENTIAL FORMS ON ALGEBRAIC VARIETIES

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Introduction. In the book "Foundations of algebraic geometry"[1] A. Weil proposed the following problem; *does every differential form of the first kind on a complete variety* **U** *determine on every subvariety* **V** *of* **U** *a differential form of the first kind?* This problem was solved affirmatively by S. Koizumi when **U** is a complete variety without multiple point.[2] In this note we answer this problem in affirmative in the case where **V** is a simple subvariety of a complete variety **U** (in §1). When the characteristic is 0 we may extend our result to a more general case but this does not hold for the case characteristic $p \neq 0$ (in §2).

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§1. Let $K = k(x_1, \ldots, x_N) = k(x)$ be a field, generated over a field k by a set of quantities (x), the class $\mathfrak{P}$ of equivalent $(n-1)$-dimensional valuations for K/k is called a prime divisor in the sense of Zariski,[3] n being the dimension of K over k, and its normalized valuation with rational integers as the value group is denoted by $\nu_{\mathfrak{P}}$. Let $F(x, dx)$ be a differential form belonging to the extension $k(x)$ of k. We say that $F(x, dx)$ is *finite at* $\mathfrak{P}$ if $F(x, dx)$ is of the form

$$F(x, dx) = \sum z_{\alpha\beta\ldots} dy_\alpha dy_\beta \ldots ,$$

where $\nu_{\mathfrak{P}}(z_{\alpha\beta\ldots}) \geqq 0$, $\nu_{\mathfrak{P}}(y_\alpha) \geqq 0$, $\nu_{\mathfrak{P}}(y_\beta) \geqq 0, \ldots$.

THEOREM 1. *Let* $\mathbf{U}^n$ *be a complete variety and* k *a field of definition of* $\mathbf{U}^n$ *which is perfect. Let* $\mathbf{P}$ *be a generic point of* $\mathbf{U}^n$ *over* k. *Then, for every differential form* ω *on* **U** *defined over* k, $\omega(\mathbf{P})$ *is of the first kind if and only if it is finite at every prime divisor of* $k(\mathbf{P})$.

Proof. Sufficiency. Let (y) be a set of quantities such that $k(\mathbf{P}) = k(y)$ and let P' be a simple point of the locus V^n of (y) over k. If P^* is a generic

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[1] We refer this book by F in this note.

[2] S. Koizumi, *On the differential forms of the first kind on algebraic varieties.* I. Journal of the Mathematical Society of Japan, vol. 1 (1949). II. vol. 2 (1951).

[3] See O. Zariski, *The reduction of the singularities of an algebraic surface.* Annals of Math. vol. 40 (1939).