# ON THE IMBEDDING PROBLEM OF NORMAL ALGEBRAIC NUMBER FIELDS

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Let $G$ and $H$ be finite groups. If a group $\bar{G}$ has an invariant subgroup $\bar{H}$, which is isomorphic with $H$, such that the factor group $\bar{G}/\bar{H}$ is isomorphic with $G$. then we say that $\bar{G}$ is an extension of $H$ by $G$. Now let $G$ be the Galois group of a normal extension $K$ over an algebraic number field $k$ of finite degree. The imbedding problem concerns us with the question, under what conditions $K$ can be imbedded in a normal extension $L$ over $k$ such that the Galois group of $L$ over $k$ is isomorphic with $\bar{G}$ and $K$ corresponds to $\bar{H}$. Brauer connected this problem with the structure of algebras over $k$, whose splitting fields are isomorphic with $K$. Following his idea, Richter investigated its local aspect using the norm theorem in the class field theory. Considering the case, where $G$ is a $p$-group and the order of $H$ is $p$, Scholz, Reichardt, and Tannaka succeeded to construct a normal extension over $k$, whose Galois group is a given $p$-group with $p \neq 2$. Scholz also solved the case, where $G$ and $H$ are both abelian. In spite of the efforts of these mathematicians the general case remains in a situation very difficult to approach. In the present paper we shall investigate the case, where $G$ is arbitrary and $H$ abelian of type $(p, \ldots, p)$ for a prime number $p$. In view of the fact, that every solvable group has a chief series $\{G_i\}$ such that the factor groups $G_i/G_{i+1}$ are abelian of type $(p, \ldots, p)$, the following investigation shall be available for the construction of normal extensions with solvable groups.

In the following we identify $\bar{H}$ with $H$. Let $g_s \in \bar{G}$ be a representative of the coset, which corresponds to $s \in G$. We denote with $sh$ the element $g_s h g_s^{-1} \in H$, which is uniquely determined for $s \in G$ and $h \in H$ irrespective of the choice of $g_s$ from the coset. $H$ becomes a $G$-module by this operation and yields a representation $\Lambda$ of $G$. If the rank of $H$ is $n$, then every element in $H$ can be regarded as an $n$-dimensional vector, whose components are integers mod. $p$. If it corresponds a matrix $\Lambda(s)$ for $s \in G$ in the representation $\Lambda$, then $sh = \Lambda(s)h$. From $g_s g_t = A(s, t) g_{st}$ with $A(s, t) \in H$ it follows

$$(1) \qquad A(s, t) + A(st, u) = A(s, tu) + \Lambda(s) A(t, u),$$

where $A(s, t)$ is called *the factor set of the extension* $\bar{G}$ of $H$ by $G$. If we take $g'_s = B(s) g_s$ with $B(s) \in H$ in place of $g_s$, then we have a factor set $A'(s, t)$, which

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