SOME THEOREMS ON OPEN RIEMANN SURFACES

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Let F be an open Riemann surface spread over the z-plane. We say that F is of positive or null boundary, according as there exists a Green's function on F or not. Let $u(z)$ be a harmonic function on F and

$$D(u)=\iint_F\left(\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right)dxdy \qquad (z=x+iy)$$

be its Dirichlet integral. As R. Nevanlinna[1] proved, if F is of null boundary, there exists no one-valued non-constant harmonic function on F, whose Dirichlet integral is finite. This Nevanlinna's theorem was proved very simply by Kuroda.[2] By this mehod, we will prove

THEOREM 1. *Let F be an open Riemann surface with null boundary and Δ be a non-compact domain on F, whose houndary Λ consists of (compact or non-compact) analytic curves. Let $u(z)$ be a one-valued harmonic function in Δ, such that $u(z)=0$ on Λ and its Dirichlet integral in Δ*

$$D(u)=\iint_\Delta\left(\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right)dxdy$$

is finite. Then $u(z)\equiv 0$.

This theorem was proved by R. Nevanlinna[3] under the condition that $u(z)$ is harmonic outside a compact domain F_0 and its Dirichlet integral in $F-F_0$ is finite.

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1) (a) R. Nevanlinna: Quadratisch integrierbare Differentiale auf einer Riemannschen Mannigfaltigkeit. Annales Acad. Sci. Fenn. Series A, Mathematica-Physica **1** (1941).
(b) Über das Anwachsen des Dirichletintegrals einer analytischen Funktion auf einer offenen Riemannschen Fläche. Annales Acad. Sci. Fenn. Series A, Mathematica-Physica **45** (1948).

2) T. Kuroda: Some remarks on an open Riemann surface. To appear in the Tohoku Math. Journ.

3) R. Nevanlinna. l.c. 1) (a).