CHARACTERIZATION OF CERTAIN RIEMANN SPACES BY DEVELOPMENT

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The purpose of this paper is to characterize the Riemann space whose line element is given by

(1) $$ds^2 = a(x^i)^2 g_{\alpha\beta}(x^\gamma)dx^\alpha dx^\beta + g_{\lambda\mu}(x^i)dx^\lambda dx^\mu.$$

Through the whole description we use the indices $i, j, \alpha, \beta, \gamma, \lambda, \mu, \nu$ when they run as follows

$$i, j \quad = 1, 2, \ldots, n$$
$$\alpha, \beta, \gamma = 1, 2, \ldots, k$$
$$\lambda, \mu, \nu = k+1, k+2, \ldots, n$$

and $a(x^i)$ and $g_{\lambda\mu}(x^i)$ shall mean to be functions of $x^1, \ldots, x^n$, $g_{\alpha\beta}(x^\gamma)$ a function of $x^1, \ldots, x^k$ and $g_{\lambda\mu}(x^\nu)$ a function of $x^{k+1}, \ldots, x^n$. This Riemann space (1) contains as special cases many important spaces such as a directly decomposable space, a conformally separable space and a space with torse-forming vector field. K. Yano [1] characterized a conformally separable Riemann space by umbilical surfaces contained in it and the proof of Theorem 1 of [1] leads to the following:

THEOREM. *The necessary and sufficient condition for an n-dimensional Riemann space to have an arc-element given by (1) is that it has $n-k$-parametric family of k-dimensional totally umbilic surfaces and k-parametric family of $n-k$-dimensional surfaces which are orthogonal to the former.*

We characterize this space (1) from another point of view.

1. We begin by determining the Riemannian connection, namely the euclidean connection without torsion, attached to (1). We can take k Pfaffians

(2) $$\omega_\alpha = p_{\alpha\beta}(x^\gamma)dx^\beta$$

such that

$$g_{\alpha\beta}(x^\gamma)dx^\alpha dx^\beta = \sum(\omega_\alpha)^2.$$

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