

OPEN RIEMANN SURFACE WITH NULL BOUNDARY

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1. Recently the writer has obtained some results concerning meromorphic or algebroidal functions with the set of essential singularities of capacity zero,¹⁾ with an aid of a theorem of Evans.²⁾ In the present paper, suggested from recent interesting papers of Sario³⁾ and Pfluger,⁴⁾ the writer will extend his results to single-valued analytic functions defined on open abstract Riemann surfaces with null boundary in the sense of Nevanlinna,⁵⁾ using a lemma instead of Evans' theorem.

2. Let F be an arbitrary open Riemann surface of finite or infinite genus and $\{F_n\}$ ($n=0, 1, \dots$) be a sequence of compact domains of F which satisfies the following conditions:

- i) F_0 is simply connected,
- ii) the boundary Γ_n of F_n consists of a finite number of simple closed analytic curves,
- iii) $\bar{F}_n \subset F_{n+1}$ ($n=0, 1, \dots$) where \bar{F}_n denotes the closure of F_n ,
- iv) every component of the open set $F - \bar{F}_n$ consists of a finite number of non-compact domains,

$$\text{v) } \bigcup_{n=0}^{\infty} F_n = F.$$

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¹⁾ K. Noshiro: [1] Contributions to the theory of the singularities of analytic functions, Jap. Journ. of Math. **19** (1948), pp. 299-327; [2] Note on the cluster sets of analytic functions, Journ. Math. Soc. Japan, **1** (1950), pp. 275-281; [3] A theorem on the cluster sets of pseudo-analytic functions, Nagoya Math. Journ. **1** (1950), pp. 83-89.

²⁾ G. C. Evans: Potentials and positively infinite singularities of harmonic functions, Monatshefte für Math. und Phys. **43** (1936), pp. 419-424.

³⁾ Leo Sario: [1] Über Riemannsche Flächen mit hebbarem Rand, Ann. Acad. Sci. Fenn. A. I. **50** (1948), 79 pp.; [2] Sur les problèmes du type des surfaces de Riemann, Comptes Rendus, Paris, **229** (1949), pp. 1109-1111; [3] Questions d'existence au voisinage de la frontière d'une surface de Riemann, Comptes Rendus, Paris, **230** (1950), pp. 269-271.

⁴⁾ A. Pfluger: Über das Anwachsen eindeutiger analytischer Funktion auf offenen Riemannschen Fläche, Ann. Acad. Sci. Fenn. A. I. **64** (1949), 18 pp.

⁵⁾ R. Nevanlinna: Quadratisch integrierbare Differentiale auf einer Riemannschen Mannigfaltigkeit, Ann. Acad. Sci. Fenn. A. I. **1** (1941), 34 pp.