ON A FORMULA CONCERNING STOCHASTIC DIFFERENTIALS

KIYOSI ITÔ

In his previous paper [1][1] the author has stated a formula[2] concering stochastic differentials with the outline of the proof. The aim of this paper is to show this formula in details in a little more general form (Theorem 6).

1. Definitions. Throughout this paper we assume that all stochastic processes[3] $\xi(t, \omega)$, $\eta(t, \omega)$, $a(t, \omega)$, $b(t, \omega)$, etc. are measurable in variables t and ω. A system of r one-dimensional Brownian motions independent of each other is called an r-dimensional Brownian motion.

Given two system of stochastic processes:

$$\xi=\{\xi_\lambda(t, \omega),\ \lambda\in\Lambda\}, \qquad \eta=\{\eta_\mu(t, \omega),\ \mu\in M\}. \tag{1.1}$$

We say that ξ has the property α with regard to η in $u\leqq t\leqq v$, if, for any t, the following two systems of random variables are independent of one another:

$$\begin{cases}\varphi_t=\{\xi_\lambda(\tau, \omega),\ \lambda\in\Lambda,\ \eta_\mu(\tau, \omega),\ \mu\in M,\ u\leqq\tau\leqq t\} \\ \psi_t=\{\eta_\mu(\sigma, \omega)-\eta_\mu(t, \omega),\ \mu\in M,\ t\leqq\sigma\leqq v\}.\end{cases} \tag{1.2}$$

Now we shall state an outline[4] of a stochastic integral of the form:

$$\int_s^t\xi(\tau, \omega)d\beta(\tau, \omega), \qquad u\leqq s\leqq t\leqq v, \qquad \omega\in\Omega_1, \tag{1.3}$$

where $\beta(t, \omega)$ is a one-dimensional Brownian motion and Ω_1 is a measurable subset of Ω. We shall set the two conditions on ξ;

(C. 1) $\quad$ *$\xi(t, \omega)$ has the property α concerning $\beta(t, \omega)$ in $u\leqq t\leqq v$,*

(C. 2) $\quad$ $\int_u^v\xi(\tau, \omega)^2d\tau$ *for almost all* $\omega\in\Omega_1$.

Received April 16, 1951.

[1] The number in [ ] refers to the Reference at the end of this paper.

[2] Theorem 1.1 in [1].

[3] In the analytical theory of probability any stochastic process is expressed as a function of the time parameter t and the probability parameter ω which runs over a probability space $\Omega(P)$, P being the probability distribution.

[4] Cf. [2] concerning the details.