ON A HOPF HOMOTOPY CLASSIFICATION THEOREM

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There are various generalizations of Hopf's brilliant theorem, which may be stated, as newly formulated by Alexandroff; all the homotopy classes of the mappings of a compact Hausdorff space X with $\dim X \leqq n$ into an n-sphere S^n are in a $(1-1)$-correspondence with the elements of the n-dimensional Čech cohomology group $H^n(X)$ with integer coefficients.

The object of the present work is to build up a generalization of Hopf's theorem. Let X be a compact Hausdorff space with $\dim X \leqq n$ and let Y be a connected absolute neighbourhood retract satisfying $\pi_r(Y)=0$ for each $r<n$. Making use of Hu's bridge operation introduced recently, addition can be defined in the homotopy classes of mappings of X into Y, so that the set of all the homotopy classes forms a group $\tilde{\mathfrak{H}}_n(X)$. It is also shown that this group is isomorphic to the n-th Čech cohomology group $H^n(X, \pi_n(Y))$ of X with coefficient group $\pi_n(Y)$.

1. Let A be an n-dimensional finite geometric complex, whose r-skelton, for $r \leqq n$, is usually designated by A^r, and let Y be an arcwise connected topological space with $\pi_r(Y)=0$ for each $r<n$. The set Ω of all the mappings of X into Y are seperated by the homotopy concept into the mutually disjoint homotopy classes, each of which contains at least one normal mapping f such that $f(X^{n-1})=y_0$, a fixed point of Y. Throughout the present paper mappings are assumed to be normal.

2. The simplest case where the n-th homotopy group $\pi_n(Y)$ $(n>1)$ of Y has a finite base, each element of which is free.

Let us denote a base of $\pi_n(Y)$ by $\{\alpha_1, \dots, \alpha_\lambda\}$ and denote a normal mapping by $f:(A, A^{n-1}) \to (Y, y_0)$. Then we have a characteristic cocycle $c^n(f) = \sum_i (f, \sigma_i^n)\sigma_i^n$ such that $(f, \sigma_i^n) = \sum_{j=1}^{\lambda} r_{ij}\alpha_j$, where r_{ij} is an integer. Considering a complex $P^n = S_1^n \vee S_2^n \vee \dots \vee S_\lambda^n$ constructed by joining n-dimensional spheres S_i^n $(i=1 \dots \lambda)$ at a point $*$, we define a mapping $h:(P^n, *) \to (Y, y_0)$ such

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