ON THE CHARACTERS OF SOLUBLE GROUPS

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The theory of representations of finite groups, which was originated by G. Frobenius, has been developed by I. Schur to become popular and studied more and more profoundly by R. Brauer even in the current stage of modern algebra. However, it seems that its applications to the structure theory of finite groups are still far from being satisfactory, partially because these two theories for structure and for representation are not firmly tied up.

In this paper I want to clarify the relationship of the structure and the representation theory in the case of soluble groups, though the case may be rather trivial.

The results are described as follows;

Let p be a fixed prime number and we use in the following modular terminologies for this p. After R. Brauer we say that a class of conjugate elements in a group is of defect d if the order of the centralizer of the element of the class is divisible exactly by p^d and that a block of characters in a group is of defect d if the degrees of all the characters of the block is divisible by p^{a-d} and at least one of them is not divisible by p^{a-d+1}, where p^a is the highest power of p which divides the group order.

In § 1 we shall prove that there exists a block of characters of defect 0 in a soluble group G, if G has no normal p-subgroup which is distinct from $\{e\}$ and has no group of the first kind as an associated group (as for definitions, see below). Since the order of the group of the first kind is even, the last condition is always satisfied by a soluble group of odd order. Further since a p-Sylow subgroup of the group of the first kind is irregular in the sense of P. Hall, the last condition is also always satisfied by a soluble group whose p-Sylow subgroup is regular. In § 2 we shall treat the case of positive defects and prove the similar theorems.

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§ 1.

We refer to an absolutely irreducible ordinary character simply as a character.

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