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A NOTE ON HOLOMORPHIC VECTOR BUNDLES OVER QUOTIENT MANIFOLDS WITH RESPECT TO NILPOTENT GROUPS

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1. A holomorphic vector bundle E over a complex analytic manifold $\mathscr{D}$ is said to be simple, if its global endomorphism ring $\mathrm{End}_C(E)$ is isomorphic to C. Projectifying the fibers of E, we get the associated projective bundle $P(E)$ of E. If we can choose a system of constant transition functions of $P(E)$, the projective bundle $P(E)$ is said to be locally flat.

In the present note we shall prove the following the theorem:

THEOREM 1. *Let Γ be a finitely generated nilpotent subgroup in the group of automorphisms of a complex analytic manifold $\mathscr{D}$. Assume that Γ acts properly discontinuously on $\mathscr{D}$ without fixed points. Let E be a holomorphic vector bundle over the quotient manifold $\mathscr{D}/\Gamma$ such that* i) *the inverse image of E with respect to the natural map $\mathscr{D} \to \mathscr{D}/\Gamma$ is trivial,* ii) *the associated projective bundle $P(E)$ is locally flat and* iii) *E is simple. Then there exists a subgroup Δ of finite index in Γ and a line bundle L over the quotient $\mathscr{D}/\Delta$ such that E is isomorphic to the direct image of L with respect to the natural map $\mathscr{D}/\Delta \to \mathscr{D}/\Gamma$.*

A complex nilmanifold is defined as the quotient of simply connected nilpotent complex Lie group G with respect to a discrete subgroup Γ of G. The finiteness of $\dim G$ implies the finite generation of Γ, and G is biholomorphic to a complex vector space. Hence, applying Theorem 1 to $\mathscr{D} = G$, we conclude that

THEOREM 2. *Let Γ be a discrete subgroup in a simply connected nilpotent complex Lie group G. Let E be a holomorphic vector bundle*

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