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BOUNDARY BEHAVIOUR OF FUNCTIONS WITH HADAMARD GAPS

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1. Introduction. In this paper we discuss the boundary properties of a function f which is analytic in the open unit disc Δ and has Hadamard gaps—i.e.

$$f(z) = \sum_{n=0}^{\infty} a_n z^{\lambda_n} \qquad (z \in \Delta) \tag{1}$$

where

$$\frac{\lambda_{n+1}}{\lambda_n} \geq q > 1 \qquad (n = 1, 2, 3, \cdots)\,. \tag{2}$$

This gap structure inhibits the possibility of cancellation in the Taylor series. Indeed, M. Weiss [11] has shown that, under appropriate growth conditions on the coefficients, the partial sums of the series (1) behave like independent random variables on $\partial\Delta$ and, in particular, that a law of the iterated logarithm holds.

The behaviour of f must therefore be expected to be highly irregular all around the boundary of the unit disc. In this sense, the boundary properties of functions with Hadamard gaps are 'typical' of the boundary properties of 'almost all' functions analytic in Δ. (See Offord [9].)

Suppose firstly that $\sum |a_n| < \infty$. Then f is continuous on $\bar{\Delta}$ and there is nothing more to be said. We exclude this trivial case and assume from now on that

$$\sum |a_n| = \infty\,. \tag{3}$$

Under this hypothesis, $\partial\Delta$ is a natural boundary for f (Ostrowski [10]). One may ask, what is the range of f? Presumably this is the whole of the complex plane covered infinitely often. However, in the

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