K. Sato
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CORES OF POTENTIAL OPERATORS FOR PROCESSES WITH STATIONARY INDEPENDENT INCREMENTS

KEN-ITI SATO

1. Introduction.

Let $X_t(\omega)$ be a stochastic process with stationary independent increments on the N-dimensional Euclidean space R^N, right continuous in $t \geqq 0$ and starting at the origin. Let $C_0(R^N)$ be the Banach space of real-valued continuous functions on R^N vanishing at infinity with norm $\|f\| = \sup_x |f(x)|$. The process induces a transition semigroup of operators T_t on $C_0(R^N)$:

$$T_t f(x) = Ef(x + X_t) .$$

The semigroup is strongly continuous. Let A be the infinitesimal generator of the semigroup, and J_λ, $\lambda > 0$, be the resolvent. The potential operator V in Yosida's sense [7] is defined by $Vf = \lim_{\lambda \to 0+} J_\lambda f$ (limit in the strong topology) if and only if the set of f for which the limit exists is dense. If V is defined, then A is one-to-one, $V = -A^{-1}$, and hence V is a closed operator (see [7] or [4]). It is proved in [4] that the semigroup T_t admits a potential operator except if $X_t = 0$ with probability one. A subset $\mathfrak{M}$ of $\mathfrak{D}(V)$ is called a core of V, if for each $f \in \mathfrak{D}(V)$ there is a sequence $\{f_n\}$ in $\mathfrak{M}$ such that $f_n \to f$ and $Vf_n \to Vf$ strongly. The purpose of this paper is to describe cores of the potential operator V. An importance of finding cores of V lies in the fact that the operator V considered only on a core is enough to determine the semigroup. That is, if two strongly continuous semigroups $T_t^{(1)}$ and $T_t^{(2)}$ have potential operators $V^{(1)}$ and $V^{(2)}$, respectively, and if $V^{(1)}$ and $V^{(2)}$ coincide on a common core, then $T_t^{(1)}$ and $T_t^{(2)}$ are identical.

Let Σ be the collection of points x such that for each open neigh-

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