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Φ-BOUNDED HARMONIC FUNCTIONS AND THE CLASSIFICATION OF HARMONIC SPACES

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1. By a *harmonic space* we mean a pair (X, H) where X is a locally compact, non-compact, connected, locally connected Hausdorff space; and H is a sheaf of *harmonic functions* defined as follows: Suppose to each open set $\Omega \subset X$ there corresponds a linear space $H(\Omega)$ of finitely-continuous real-valued functions defined on Ω. Then $H = \{H(\Omega)\}_\Omega$ must satisfy the three axioms of Brelot (1) and in addition Axiom 4 of Loeb (4): 1 is H-superharmonic in X.

Denote by $\Phi(t)$ a nonnegative real-valued function defined on $[0, \infty)$. We stress that except for the condition $\Phi(t) \geqslant 0$ nothing is required of $\Phi(t)$ such as continuity and measurability. A harmonic function u on X (when H is well-understood we simply refer to X itself as the harmonic space) is called *Φ-bounded* if the composite function $\Phi(|u|)$ possesses a harmonic majorant on X. The notion of Φ-boundedness is due to Parreau (9) who considered the special case of an increasing, convex Φ. Later Nakai (6), using general Φ, completely determined the class $O_{H\Phi}$ of Riemann surfaces for which every Φ-bounded harmonic function reduces to a constant. Recently Ow (8) considered the classification of harmonic spaces with respect to Φ-bounded harmonic functions using a stronger assumption that Loeb's Axiom 4; namely it was assumed that $1 \in H$.

Since the case $1 \in H$ has already been considered, as mentioned above, throughout this paper we will make the following assumption:

$$1 \notin H .$$

This condition occurs, for example, in the study of the harmonic space of solutions of the elliptic partial differential equation $\Delta u = Pu$, where $P \not\equiv 0$ is a nonnegative function on a manifold X.

The main object of this paper is to show that in view of the con-

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