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Nagoya Math. J.
Vol. 47 (1972), 29–48

STABLE VECTOR BUNDLES ON ALGEBRAIC SURFACES

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Let k be an algebraically closed field, and X a nonsingular irreducible projective algebraic variety over k. These assumptions will remain fixed throughout this paper. We will consider a family of vector bundles on X of fixed rank r and fixed Chern classes (modulo numerical equivalence). Under what condition is this family a bounded family? When X is a curve, Atiyah [1] showed that it is so if all elements of this family are indecomposable. But when X is a surface, he showed also that this condition is not sufficient. We give the definition of an H-stable vector bundle on a variety X. This definition is a generalization of Mumford's definition on a curve. Under the condition that all elements of a family are H-stable of rank two on a surface X, we prove that the family is bounded. And we study H-stable bundles, when X is an abelian surface, the projective plane or a geometrically ruled surface.

1. H-stable vector bundles
2. H-stable vector bundles on algebraic surfaces
3. H-stable vector bundles on geometrically ruled surfaces
4. Simple vector bundles on the projective plane
5. Stable vector bundles on abelian surfaces.

1. H-stable vector bundles.

In this paper, we use the words vector bundles and locally free sheaf of finite rank interchangeably. Let F be a coherent sheaf on X. Under our hypothesis on X, we can define an invertible sheaf $\mathrm{Inv}(F)$ (first Chern class cf. [5]), i.e. let $E.$ be a finite resolution of F by locally free sheaves E_i. $\mathrm{Inv}(F) = \bigotimes_i (\dot{\wedge} E_i)^{(-1)^i}$ where $\dot{\wedge}$ denotes the highest exterior power. Then $\mathrm{Inv}(F)$ depends only on F, up to canonical isomorphism. $\mathrm{Inv}(F)$ has the following properties:

Received June 28, 1971.