G. Shimura
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ON THE FIELD OF RATIONALITY FOR AN ABELIAN VARIETY

GORO SHIMURA

The purpose of this paper is to prove the following two facts:

(I) *Every generic polarized abelian variety of odd dimension has a model rational over its field of moduli.*

(II) *No generic principally polarized abelian variety of even dimension has a model rational over its field of moduli.*

In both statements and throughout the paper, we assume that the universal domain is of characteristic 0. We call a polarized abelian variety *generic* if its field of moduli has the maximum transcendence degree (i.e., $n(n+1)/2$ if the variety is of dimension n) over the rational number field.

It is well-known that an elliptic curve has a model rational over its field of moduli. However, no general result, not even a counter-example, seems to have been obtained in the higher-dimensional case. In a previous paper [5], we have shown that a polarized abelian variety with sufficiently many complex multiplications, under a certain condition, has a model rational over its field of moduli. We discuss here the other extreme case in which varieties are generic. A negative answer similar to (II) will be given also for abelian varieties with a certain type of polarization which is not necessarily principal, and for hyperelliptic curves of even genera.

It is still an open question to obtain a criterion under which an arbitrarily given polarized abelian variety has a model rational over its field of moduli. The above two statements combined together seem to indicate a rather complicated nature of the problem, which almost defies conjecture. A new viewpoint is certainly necessary to understand the whole situation.

Since the proofs are not so long, we do not try to explain the main ideas at this point, except the following two general remarks.

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