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STABLE MANIFOLDS OF A MAP AND A FLOW FOR A COMPACT MANIFOLD

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§ 0. Introduction

The purpose of this paper is to generalize the notion of the stable manifolds in Smale [5] and [6], in which the stable manifolds of flows or diffeomorphisms for a singular point or a closed orbit are defined in certain conditions. This generalization is concerned with Fenichel [1]. He considers the stable manifolds of flows and diffeomorphisms for a torus. Here, we consider the case of a compact manifold. But our argument does not exactly imply Fenichel's result.

It is interesting to investigate the conditions for the existence of stable manifolds of flows or diffeomorphisms. If the stable manifolds exist, then we can see to some extent the state of the orbits of flows or diffeomorphisms near the stable manifolds.

In § 1, we prove Theorem 1 by the method of successive approximations and we obtain a local stable manifold of a map for a compact manifold as a graph of the solution map. In Corollary of Theorem 1, we study the state of the orbits of a map. In § 2, we construct a local stable manifold of a flow by using the result of § 1 and we study the state of the orbits of the flow.

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§ 1. The stable manifold of a map.

First, we shall explain the notations.

Let T be a compact C^l-manifold $(1 \leq l < \infty)$ and E_i be a k_i dimensional Euclidean space for $i = 1, 2$. We denote by L_i a $k_i \times k_i$ non-singular matrix for $i = 1, 2$. Define the norm of a vector $z = (z_1, \cdots, z_{k_i}) \in E_i$ (resp. a matrix L_i) by $\|z\| = \max(|z_1|, \cdots, |z_{k_i}|)$ (resp. $\|L_i\| = \sup_{\|z\|=1} \|L_i z\|$). We suppose

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