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FUNCTIONAL EQUATIONS OF GENERALIZED EPSTEIN ZETA FUNCTIONS IN SEVERAL COMPLEX VARIABLES

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Let $S^{(n)}$ be the matrix of a positive definite quadratic form and $(\rho_1, \cdots, \rho_{r-1}) \in \boldsymbol{C}^{r-1}$. Define

$$\zeta_{n_1,\cdots,n_r}(S, \rho_1, \cdots, \rho_{r-1}) = \sum \prod_{i=1}^{r-1} |S[U_i]|^{-\rho_i}. \tag{1}$$

Here the sum is over unimodular matrices $U^{(n)} = (U_i^{(n,N_i)} *)$ which lie in a complete set of representatives for the equivalence relation $U \sim V$ if $U = VP$, with P unimodular and having block form

$$P = \begin{pmatrix} P_1^{(n_1)} & & * \\ & \ddots & \\ 0 & & P_r^{(n_r)} \end{pmatrix}.$$

The following notation shall be used throughout:

$A^{(n,m)}$ for an n by m matrix A

$A^{(n)} = A^{(n,n)}$

$|A| =$ determinant of A

$S[A] = {}^tASA$, ${}^tA$ being the transpose of A

$N_i = \sum_{j=1}^{i} n_j, \ i = 1, 2, \cdots, r, \ N_r = n = \sum_{j=1}^{r} n_j.$

A unimodular matrix $U^{(n)}$ is one with integral entries and determinant ± 1.

The function (1) is clearly a generalization of Epstein's zeta function, as well as a generalization of functions defined by Koecher [1], Maass [3], and Selberg [4], [5], [6]. It can also be viewed as an Eisenstein series; for which, see Langlands [2].

If $n_i = 1$ for all $i = 1, 2, \cdots, r$ and $r = n$, denote the function defined by (1) as $\zeta_{(n)} = \zeta_{1,\cdots,1}$.

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