Y. Tsushima.
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ON THE BLOCK OF DEFECT ZERO

YUKIO TSUSHIMA

1. Let G be a finite group and let p be a fixed prime number. If D is any p-subgroup of G, then the problem whether there exists a p-block with D as its defect group is reduced to whether $N_G(D)/D$ possesses a p-block of defect 0. Some necessary or sufficient conditions for a finite group to possess a p-block of defect 0 have been known (Brauer-Fowler [1], Green [3], Ito [4] [5]). In this paper we shall show that the existences of such blocks depend on the multiplicative structures of the p-elements of G. Namely, let $\mathfrak{p}$ be a prime divisor of p in an algebraic number field which is a splitting one for G, $\mathfrak{o}$ the ring of $\mathfrak{p}$-integers and $k = \mathfrak{o}/\mathfrak{p}$, the residue class field. Then,

THEOREM 1. *Let c denote the sum of the p-elements of G including the identity in the group ring kG. Then c^2 is equal to the sum of the block idempotents of p-defect* 0.

2. Proof of the Theorem and Corollaries

First we note,

LEMMA 1. *Let A be a quasi Frobenius algebra with minimum conditions and let B be a block of A, i.e. a two sided direct summand of A which is indecomposable. If there exists a primitive idempotent e in B such that Ae is a minimal left ideal, then B is a simple algebra.*

Proof. Let f be any primitive idempotent in B. It suffices to show that $Ae \cong Af$ by assuming Ae and Af have a composition factor in common, since e and f are linked ([2] § 55). Then there exists an injection $0 \to Ae \to Af/M$ for a suitable A-submodule M of Af. But since Ae is injective, this map splits. Therefore there exists an epimorphism $Af \to Ae \to 0$, which

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