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RIEMANN DOMAINS WITH BOUNDARY OF CAPACITY ZERO

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§ 1. Introduction.

The well-known Thullen-Remmert-Stein's theorem ([9], [7]) asserts that, for a domain D in $\boldsymbol{C}^N$ and an n-dimensional irreducible analytic set S in D, a purely n-dimensional analytic set A in $D-S$ has an essential singularity at any point in S if A has at least one essential singularity in S. In [1], E. Bishop generalized this to the case that A has the boundary of capacity zero in his sense. Afterwards, in [8], W. Rothstein obtained more precise informations on the essential singularities of A under the assumption $\dim A = 1$. The main purpose in this paper is to generalize these Rothstein's results to the case of arbitrary dimensional analytic sets.

We consider a Riemann domain (X, π, M) with boundary of capacity zero, namely, a triple of a connected n-dimensional normal complex space X, a connected n-dimensional complex manifold M and a discrete holomorphic map $\pi : X \to M$ with the following properties:

For any $z_0 \in M$ there are a neighborhood U of z_0 and a plurisubharmonic function $u(x)$ on $\pi^{-1}(U)$ such that (i) $u(x) \leqq 0$, (ii) $u(x) \not\equiv -\infty$ on any connected component of $\pi^{-1}(U)$ and (iii) $\lim_{\nu\to\infty} u(x_\nu) = -\infty$ for any sequence $\{x_\nu\}$ without accumulation points in X if $\lim_{\nu\to\infty} \pi(x_\nu)$ exists in U.

The first main result is the following

THEOREM I. *If (X, π, M) is a Riemann domain with boundary of capacity zero, then $M - \pi(X)$ is of capacity zero* (c.f. Definition 2.5).

We define a direct boundary point of a Riemann domain on the analogy of a direct transcendental singularity in the theory of functions of one complex variable (c.f. Definition 4.3). As an application of Theorem I, we give the following theorem, which is a generalization of a result in [4].

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