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CHARACTERISTIC CLASSES FOR PL MICRO BUNDLES

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§ 0. Introduction.

Let $BSPL$ be the classifying space of the stable oriented PL micro bundles. The purpose of this paper is to determine $H_*(BSPL : Z_p)$ as a Hopf algebra over Z_p, where p is an odd prime number. In this chapter, p is always an odd prime number.

The conclusions are as follows.

THEOREM 2–22. *As a Hopf algebra over* Z_p, $H_*(BSPL : Z_p) = Z_p[\bar{\bar{b}}_1, \bar{\bar{b}}_2, \cdots] \otimes Z_p[\sigma(\bar{\bar{x}}_I)] \otimes \Lambda(\sigma(\bar{\bar{x}}_J))$. $\Delta(\bar{\bar{b}}_j) = \sum_{i=0}^{j} \bar{\bar{b}}_i \otimes \bar{\bar{b}}_j$, $b_0 = 1$, $\sigma(\bar{\bar{x}}_I)$, $\sigma(\bar{\bar{x}}_J)$ *are primitive.*

THEOREM 3–1. *As a Hopf algebra over* $Z[1/2]$,

i) $H^*(BSPL : Z[1/2])/_{Torsion} = Z[1/2][R_1, R_2, \cdots]$

ii) $\Delta R_j = \sum_{i=0}^{j} R_i \otimes R_{j-i}$, $R_0 = 1$. $deg\, R_j = 4j$.

iii) *In* $H^*(BSPL : Q) = Q[p_1, p_2, \cdots]$, R_j *are expressed as follows.*

$$R_j = 2^{a_j}(2^{2j-1} - 1)\, Num\,(B_j/4j) \cdot p_j + dec, \text{ for some } a_j \in Z.$$

Let $MSPL$ denote the spectrum defined by the Thom complex of the universal PL micro bundle over $BSPL(n)$, and $A = A_p$ denote the mod p Steenrod algebra. And $\phi : A \to H^*(MSPL : Z_p)$ is defined by $\phi(a) = a(u)$, where $u \in H^0(MSPL : Z_p)$ is the Thom class.

THEOREM 4–1. *The kernel of* ϕ *is* $A(\underline{\underline{Q}}_0, \underline{\underline{Q}}_1)$, *the left ideal generated by Milnor elements* $\underline{\underline{Q}}_0$, $\underline{\underline{Q}}_1$.

This is the conjecture of Peterson [12].

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