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COMPLEX-HARMONIC MEIER'S THEOREM

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1. Fatou's theorem is true for a bounded complex-valued harmonic function in the disk D: $|z| < 1$. One asks naturally: "Is Meier's topological analogue of Fatou's theorem (simply, "*MF* theorem"; [14, p. 330, Theorem 6], cf. [10, p. 154, Theorem 8.9]) true for a bounded complex-valued harmonic function in D?" We shall give the affirmative answer to this question. Furthermore, the horocyclic *MF* theorem [2, p. 14, Theorem 5] in the complex-harmonic form will be proved in parallel.

For recent various discussions on Plessner's and Meier's theorems we consult [1∼7, 11, 12, 15∼18].

2. In the rest of this note we denote by $\delta(\zeta_0, \rho)$ the open disk $|z - \zeta_0| < \rho$ in the z-plane.

LEMMA 1. *Let a function $g(\zeta)$ be complex-valued and harmonic (simply, "complex-harmonic") in $\delta(\zeta_0, \rho)$ and $|g(\zeta)| < 1$ for $\zeta \in \delta(\zeta_0, \rho)$. Then we have*

$$(1) \qquad |g(\zeta) - g(\zeta_0)| \leqq (8/\pi) \text{ arc } \tan(|\zeta - \zeta_0|/\rho)$$

for $\zeta \in \delta(\zeta_0, \rho)$ (Schwarz's lemma).

Proof. Let $w = (\zeta - \zeta_0)/\rho$ and consider the function

$$G(w) = \{g(\rho w + \zeta_0) - g(\zeta_0)\}/2$$

in D: $|w| < 1$. Then $G(0) = 0$ and $|G(w)| < 1$ in D, so that we may apply the ready Schwarz lemma [13, p. 101, Lemma] to the complex-harmonic G in D. The inequality [13, p. 101, (3)]

$$G(w)| \leqq (4/\pi) \text{ arc } \tan|w|$$

for $w \in D$ proves (1). Q.E.D.

The reader should know the definition of cluster set, chordal cluster set and angular cluster set [10, pp. 1, 72 and 73].

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