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ON A CHARACTERIZATION OF THE FIRST RAMIFICATION GROUP AS THE VERTEX OF THE RING OF INTEGERS

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1. Introduction.

Let k be a $\mathfrak{p}$-adic number field and $\mathfrak{o}$ be the ring of all integers in k. Moreover, let K/k be a finite Galois extension with the Galois group $G = G(K/k)$. Then the ring $\mathfrak{O}$ of all integers in K is an $\mathfrak{o}[G]$-module. In this paper we shall give a characterization of the first ramification group G_1 of the extension K/k as the vertex of $\mathfrak{O}$ which is defined below.

To define the vertex of $\mathfrak{O}$, we remember the vertex theory ([1], [2]). Let G be an arbitrary finite group and M be an $\mathfrak{o}[G]$-module, where $\mathfrak{o}[G]$ is the group algebra of G over $\mathfrak{o}$. Let U be a subgroup of G. Then M is said to be U-projective if there is an $\mathfrak{o}[U]$-module N such that M is isomorphic to a component of the induced $\mathfrak{o}[G]$-module $\mathfrak{o}[\mathfrak{O}]\otimes_{\mathfrak{o}[U]}N$. If M is an indecomposable $\mathfrak{o}[G]$-module, then there exists a subgroup V of G, such that

(i) M is V-projective

and

(ii) if W is any subgroup of G, such that M is W-projective, then for some element g of G

$$gVg^{-1}\subset W.$$

We call such V, that is uniquely determined up to conjugate subgroups in G, a vertex of M.

Now let K/k be the finite Galois extension with the Galois group G, and $\mathfrak{O}$ the ring of all integers in K. Then, since $\mathfrak{O}$ is not always indecomposable as an $\mathfrak{o}[G]$-module, we consider the decomposition of $\mathfrak{O}$ into indecomposable $\mathfrak{o}[G]$-modules M_i

$$\mathfrak{O} = M_1 \oplus \cdots \oplus M_l. \tag{1}$$

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