Masatoshi Yamauchi
Nagoya Math. J.
Vol. 43 (1971), 137–149

ON THE TRACE OF HECKE OPERATORS FOR CERTAIN MODULAR GROUPS

MASATOSHI YAMAUCHI

Introduction.

The trace of Hecke operators with respect to a unit group of an order in a quaternion algebra has been given in Eichler [1], [2] in the case when the order is of square-free level. The purpose of this note is to study the order of type (q_1, q_2, q_3) (see text 1.1), in the case, of cube-free level, and to give a formula for the trace of Hecke operators in the case $q_3 = 2$.

Notation.

$\boldsymbol{Z}$, $\boldsymbol{Q}$, $\boldsymbol{R}$ denote the ring of rational integers, the field of rational numbers, and the field of real numbers, respectively. $\boldsymbol{Q}_p$ denotes the p-adic closure of $\boldsymbol{Q}$ and $\boldsymbol{Z}_p$ the ring of integers in $\boldsymbol{Q}_p$. R being a ring, $M_2(R)$ denotes the full matrix ring over R of degree 2.

1. The order of type (q_1, q_2, q_3)

1.1. Let A be a quaternion algebra over $\boldsymbol{Q}$ and $q_1^2 = d(A/\boldsymbol{Q})$ be its discriminant. For every prime number p, $A_p \underset{\boldsymbol{Q}}{\otimes} \boldsymbol{Q}_p$ is a division algebra over $\boldsymbol{Q}_p$ or $A_p \simeq M_2(\boldsymbol{Q}_p)$ according as $p \mid q_1$ or $p \nmid q_1$. Let q_2, q_3 be square-free positive integers such that $(q_i, q_j) = 1$ for $i \neq j$, $1 \leq i, j \leq 3$. We then define the order $\mathfrak{O}$ of type (q_1, q_2, q_3) which satisfies the following properties:

i) $\mathfrak{O}_p = \mathfrak{O} \underset{\boldsymbol{Z}}{\otimes} \boldsymbol{Z}_p$ is a maximal order in A_p, if $p \nmid q_1 q_2 q_3$,

ii) $\mathfrak{O}_p$ is the unique maximal order in the division algebra A_p, if $p \mid q_1$,

iii) $\mathfrak{O}_p \cong \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(\boldsymbol{Z}_p) \mid c \equiv 0 \pmod{p} \right\}$, if $p \mid q_2$,

iv) $\mathfrak{O}_p \cong \left\{ \begin{pmatrix} a & c \\ b & d \end{pmatrix} \in M_2(\boldsymbol{Z}_p) \mid c \equiv 0 \pmod{p^2} \right\}$, if $p \mid q_3$,

In this note we consider the order of type (q_1, q_2, q_3) exclusively.

Received August 27, 1970.
Revised November 18, 1970.