Bang-yen Chen
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SOME INTEGRAL FORMULAS FOR HYPERSURFACES IN EUCLIDEAN SPACES

BANG-YEN CHEN

1. Introduction

Let M be an oriented hypersurface differentiably immersed in a Euclidean space of $n+1 \geq 3$ dimensions. The r-th mean curvature K_r of M at the point P of M is defined by the following equation:

$$\det(\delta_{ij} + ta_{ij}) = \sum_{r=0}^{n} \binom{n}{r} K_r t^r \tag{1}$$

where δ_{ij} denotes the Kronecker delta, $\binom{n}{r} = n!/r!\,(n-r)!$, and a_{ij} are the coefficients of the second fundamental form. Throughout this paper all Latin indices take the values $1, \cdots, n$, Greek indices the values $1, \cdots, n+1$, and we shall also follow the convention that repeated indices imply summation unless otherwise stated. Let p denote the oriented distance from a fixed point 0 in E^{n+1} to the tangent hyperplane of M at the point P, and dV denote the area element of M. Let $e_1, \cdots, e_n$ be an ordered orthonormal frame in the tangent space of the hypersurface M at the point P, and denote by x_i the scalar product of e_i and the position vector $\boldsymbol{X}$ of the point P with respect to the fixed point 0 in E^{n+1}. The main purpose of this paper is to establish the following theorems:

THEOREM 1. *Let M be an oriented hypersurface with regular smooth boundary differentiably immersed in a Euclidean space E^{n+1}. Then we have*

$$\int_M p^{m-1}\boldsymbol{X}\cdot\nabla K_r dV + n\int_M p^{m-1}(K_r - K_1K_rp)dV + (m-1)\int_M p^{m-2}K_r x_i x_j a_{ij} dV \tag{2}$$
$$= \int_{\partial M} p^{m-1}K_r \boldsymbol{X}\cdot * d\boldsymbol{X}, \qquad r = 0, 1, \cdots, n-1,$$

where m is any real number, ∇K_r is the gradient of K_r, ∂M is the boundary of M and $*$ denotes the star operator.

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