

P-AMPLE BUNDLES AND THEIR CHERN CLASSES

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Introduction

In [9], Hartshorne extended the concept of ampleness from line bundles to vector bundles. At that time, he conjectured that the appropriate Chern classes of an ample vector bundle were positive, and it was hoped that there would be some criterion for ampleness of vector bundles similar to Nakai's criterion for line bundles. In the same paper, Hartshorne also introduced the notion of p -ample when the ground field had characteristic p , proved that a p -ample bundle was ample and asked if the converse were true.

In the first chapter of this paper, we will show that a p -ample bundle has positive Chern classes when the characteristic of the ground field is $p \neq 0$, and that a quotient bundle of a direct sum of ample line bundles also has positive Chern classes in any characteristic. We also give a series of polynomials in the Chern classes of a bundle E which are positive if E is ample.

The second chapter will be devoted to some criteria for a vector bundle to be ample. The final chapter gives two examples of ample vector bundles on P^2 when the characteristic of k is $p \neq 0$. The first example will be ample, but not p -ample. The second bundle E will be p -ample, but $H^1(P^2, \Gamma^{p^n}(E) \otimes F)$ will be non-zero if F is a bundle and n is large.

We fix our notation. k will denote an algebraically closed field. A variety X will be a reduced, irreducible scheme of finite type over k . If X is non-singular, $A(X)$ will denote the Chow ring modulo numerical equivalence. If X is n -dimensional and complete, we have $A^n(X)$ canonically identified with \mathbf{Z} . We will often call a locally free sheaf over X a bundle. The bundle E is defined to be ample if for any coherent sheaf F on X , $F \otimes S^n(E)$ is generated by global sections for n large. E is ample if

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