Robin Hartshorne
Nagoya Math. J.
Vol. 43 (1971), 73–89

AMPLE VECTOR BUNDLES ON CURVES

ROBIN HARTSHORNE[1]

Introduction

In our earlier paper [4] we developed the basic sheaftheoretic and cohomological properties of ample vector bundles. These generalize the corresponding well-known results for ample line bundles. The numerical properties of ample vector bundles are still poorly understood. For line bundles, Nakai's criterion characterizes ampleness by the positivity of certain intersection numbers of the associated divisor with subvarieties of the ambient variety. For vector bundles, one would like to characterize ampleness by the numerical positivity of the Chern classes of the bundle (and perhaps of its restrictions to subvarieties and their quotients). Such a result, like the Riemann-Roch theorem, giving an equivalence between cohomological and numerical properties of a vector bundle, may be quite subtle. Some progress has been made by Gieseker [2], by Kleiman [8], and in the analytic case, by Griffiths [3].

Even on a complete non-singular curve over a field k, the problem is non-trivial. A line bundle is ample if and only if its degree is positive. The degree of an ample vector bundle on a curve is positive. Any quotient of an ample vector bundle is ample, and so its degree is also positive. This leads us to the following

QUESTION: Let X be a complete non-singular curve over a field k. Let E be a vector bundle, all of whose quotient bundles (including E itself) have degree positive. Then is E ample?

In our earlier paper, we found an answer to this question for bundles of rank 2 [4, 7.6 and 7.7]. In that case E is ample provided that either

Received August 20, 1970.
Revised Decemben 11, 1970.
[1] The author gratefully acknowledges financial support from Nagoya University, Nagoya, the Tata Institute of Fundamental Research, Bombay, and the Institut des Hautes Etudes Scientifiques, Paris.