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Nagoya Math. J.
Vol. 42 (1971), 115–133

ON CANONICAL REALIZATIONS OF BOUNDED SYMMETRIC DOMAINS AS MATRIX-SPACES[1)]

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Introduction

It is the purpose of the present paper to give a natural method of realizing bounded symmetric domains as matrix-spaces. Our method yields, as special cases, the well-known bounded models of irreducible bounded symmetric domains of classical type (I)-(IV), as were already described in the original paper of E. Cartan [1] (see § 3; we follow in this paper the classification table in [14], not in [1]). A direct application of this method will be to determine the *canonical* bounded models of the irreducible bounded symmetric domains of exceptional type; it will be published in another paper (see [6], [7] for the summary of the results).

In the Appendix, we indicate briefly that our version on symmetric domains can be generalized and applied to a more general class of symmetric spaces, the so-called symmetric R-spaces of non-compact type in the sense of J. Tits; this was partly stated in Nagano [13] and Takeuchi [16].

We would like to express here our deep gratitude to M. Takeuchi who read the manuscript and suggested many improvements.

NOTATION: 1) $\boldsymbol{M}_{p,q}$ denotes the complex vector space of all complex matrices of type (p, q); in particular, we write as $\boldsymbol{M}_{p,p} = \boldsymbol{M}_p$ for brevity. Similarly $\boldsymbol{M}_{p,q}(\boldsymbol{R})$ is the real vector space of all real matrices of type (p, q).

2) $\boldsymbol{C}^n$ is the complex cartesian space of n-dimensions, and in many cases, $\boldsymbol{C}^n$ is identified with $\boldsymbol{M}_{n,1}$, or with $\boldsymbol{M}_{1,n}$.

3) For hermitian matrices A, B $(\in \boldsymbol{M}_r)$, $A < B$ means that all eigen-values of $A - B$ are negative. I_r denotes the unit matrix of degree r.

4) For complex vector spaces V, W, we denote by $\mathfrak{L}(V, W)$ the complex vector space of all complex linear mappings of V into W.

Received March 19, 1970.

1) Part of the present work was done in 1964, when the author was staying at the Institute for Advanced Study, Princeton, under the sponsorship of the National Science Foundation.