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BOUNDED ENERGY-FINITE SOLUTIONS OF $\Delta u = Pu$ ON A RIEMANNIAN MANIFOLD

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Introduction

1. The classification of Riemann surfaces with respect to the equation $\Delta u = Pu$ ($P \geq 0$, $P \not\equiv 0$) was initiated by Ozawa [13] and further developed by L. Myrberg [8, 9], Royden [14], Nakai [10, 11], Sario-Nakai [15], Nakai-Sario [12], Glasner-Katz [3], and Kwon-Sario [7].

The objective of the present paper is to establish properties of bounded energy finite solutions of $\Delta u = Pu$ in terms of the P-harmonic boundary of a Riemannian manifold R. The occurrence of the P-singular point (Nakai-Sario [12]), at which all functions in the P-algebra vanish, necessitates delicate new arguments.

The P-algebra $M_P(R)$ is not, in general, uniformly dense in the space $B(R_P^*)$ of bounded continuous functions on the P-compactification R_P^*. However, we shall prove the following Urysohn-type theorem. Let K_0, K_1 be any disjoint compact subsets of R_P^* with the P-singular point $s \in K_0$. Then there exists a function $f \in M_P(R)$ such that $0 \leq f \leq 1$ on R_P^* and $f | K_i = i$ ($i = 0, 1$).

Although the standard maximum-minimum principle does not hold, the following modification can be established. Let u be P-superharmonic and bounded from below on a Riemannian manifold R such that $\liminf u \geq 0$ at the P-harmonic boundary Δ_P. Then $u \geq 0$ on R. As a consequence, $|u| \leq \limsup_{\Delta_P} |u|$ for every bounded P-harmonic function u on R.

This maximum principle together with the orthogonal decomposition enables us to prove the existence of a positive linear operator

$$\pi: B_s(\Delta_P) \to PB(R)$$

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