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A CLASS OF RIEMANNIAN MANIFOLDS SATISFYING R(X,Y)•R = 0

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1. Introduction

Let (M, g) be a Riemannian manifold and let R be its Riemannian curvature tensor. If (M, g) is a locally symmetric space, we have

$$(*) \qquad R(X,Y)\cdot R = 0 \qquad \text{for all tangent vectors } X, Y$$

where the endomorphism $R(X,Y)$ (i.e., the curvature transformation) operates on R as a derivation of the tensor algebra at each point of M. There is a question: Under what additional condition does this algebraic condition $(*)$ on R imply that (M, g) is locally symmetric (i.e., $\nabla R = 0$)? A conjecture by K. Nomizu [5] is as follows: $(*)$ implies $\nabla R = 0$ in the case where (M, g) is complete and irreducible, and $\dim M \geq 3$. He gave an affirmative answer in the case where (M, g) is a certain complete hypersurface in a Euclidean space ([5]).

With respect to this problem, K. Sekigawa and H. Takagi [8] proved that if (M, g) is a complete conformally flat Riemannian manifold with dim $M \geq 3$ and satisfies $(*)$, then (M, g) is locally symmetric.

On the other hand, R.L. Bishop and B.O'Neill [1] constructed a wide class of Riemannian manifolds of negative curvature by warped product using convex functions. For two Riemannian manifolds B and F, a warped product is denoted by $B \times_f F$, where f is a positive C^∞-function on B. The purpose of this paper is to prove

THEOREM A. *Let (F, g) be a Riemannian manifold of constant curvature $K \leq 0$. Let E^n be an n-dimensional Euclidean space and let f be a positive C^∞-function on E^n. On a warped product $E^n \times {}_fF$, assume that*

(i) *the condition $(*)$ is satisfied, and*

(ii) *the scalar curvature is constant.*

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