Kiyoshi Shiga
Nagoya Math. J.
Vol. 42 (1971), 57–66

ON HOLOMORPHIC EXTENSION FROM THE BOUNDARY

KIYOSHI SHIGA

0. Introduction

Let D be a bounded domain of the complex n-space $C^n(n \geq 2)$, or more generally a pair (M, D) a finite manifold (c.f. Definition 2.1), and we assume the boundary ∂D is a smooth and connected submanifold. It is well known by Hartogs-Osgood's theorem that every holomorphic function on a neighbourhood of ∂D can be continued holomorphically to D. Generalizing the above theorem we shall prove that if a differentiable function on ∂D satisfies certain conditions which are satisfied for the trace of a holomorphic function on a neighbourhood of ∂D, then it can be continued holomorphically to D (Theorem 2–5). The above conditions will be called the tangential Cauchy Riemann equations.

Using the above result, we shall determine the condition for a diffeomorphism of ∂D to be continued to a holomorphic automorphism of D (Theorem 3–3). Finally as its corollary the analogy to functions holds for cross-sections of a holomorphic vector bundle. (Theorem 3–5)

In preparing this paper, I have received many advices from Professor M. Ise and Professor T. Nagano. I would like to express my cordial thanks to them.

1. Tangential Cauchy-Riemann equations

Let N be an n-dimensional complex manifold. From now on we always assume $n \geq 2$. Let M be a real smooth submanifold of N. We denote by $T_p(M)$ the real tangent space of M at p. Let J be the complex structure of N.

$$C_p = T_p(M) \cap JT_p(M)$$

is the maximum complex subspace of $T_p(M)$, and we denote its complex dimension by $m(p)$ and we assume $m(p)$ is constant on M.

Received March 13, 1970.