John A. Beekman
Nagoya Math. J.
Vol. 42 (1971), 9–21

SEQUENTIAL GAUSSIAN MARKOV INTEGRALS*

JOHN A. BEEKMAN

I. Introduction

In [6] R.H. Cameron defined and studied a sequential Wiener integral. This was motivated by the function space integral R.P. Feynman used in [12] to give a solution to the Schröedinger equation. In [5] the present author studied sequential Gaussian Markov integrals with a positive parameter. This paper gives sufficient conditions on the integrand for such integrals to exist, when the parameter is complex. These sequential integrals are related to ordianry Gaussian Markov integrals through a Fourier transform type formula extended from [5]. We shall show that such integrals are equal to conditional Wiener integrals of suitably modified functionals.

As is well-known, function space itegrals are used in many fields. We will use the sequential integrals in certain applications in physics, but it is believed they will prove useful in other areas. Specifically, we shall show that sequential integrals of appropriate functionals satisfy generalized Schröedinger equations and Dirac delta function conditions. We shall also prove that certain sequential integrals solve integral equations formally analogous to the differential equations of [5]. Our use of the word potential is the quantum mechanics use—see [5], for example.

For completeness, several references will be mentioned. References [1] through [5] consider the connection between Gaussian Markov stochastic processes and generalized Schröedinger equations. Reference [20] discusses the same connection, with heavy emphasis on the physics involved. R.H. Cameron's papers [6] through [9] have contributed much to this area.

II. Sequential Integrals

Let $\{X(\tau),\ s \leq \tau \leq t\}$ be a Gaussian Markov process with transition density function

Received May 4, 1970.
* This research was partially supported by the National Science Foundation through grants NSF GP-7639 and NSF GP-9634.