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RAMIFICATION THEORY FOR EXTENSIONS OF DEGREE p

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Introduction. The notions of tame and wild ramification lead us to make the following definition.

DEFINITION. *The quotient field extension of an extension of discrete rank one valuation rings is said to be fiercely ramified if the residue class field extension has a nontrivial inseparable part.*

The purpose of this paper is to study ramification in Galois extensions $K \supset k$ of degree p. The ground field k is the quotient field of a complete discrete rank one valuation ring R of unequal characteristic, and p denotes the characteristic of $\bar{R}$. Assume furthermore that R contains a primitive p^{th} root of unity, from which it follows that the absolute ramification index a of R is divisible by $p-1$.

Observe that a Galois extension of degree p may be unramified, wild, or fierce. In order to study the properties of such an extension relative to ramification we established a technique for computing the integral closure S of R in K.

The computation of S is facilitated by a judicious choice of the element of k whose p^{th} root defines the extension. Let $U^{(i)}$ for $i \geq 0$ denote the usual filtration on $U(R)$, and let $U^{(-1)}$ denote the set of prime elements of R. In Section 1 we associate to each Galois extension $K \supset k$ of degree p an integer x with $-1 \leq x \leq p$ called the *field exponent* of the extension such that $K = k(b^{1/p})$ for some element b of $U^{(x)}$ (see Prop. 1.6).

The ring $R[b^{1/p}]$ where b is in $U^{(x)}$ is contained in the integral closure S, but equality need not hold. In Section 2 we present a technique for computing S which entails the construction of a chain (S_i) with $0 \leq i \leq g$ of simple ring extensions S_i of R where $S_0 = R[b^{1/p}]$, $S_{i-1} \subset S_i$, and $S_g \subseteq S$. The integer g satisfies the inequality $0 \leq g \leq (a/p-1) - 1$ and is called the

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