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ON ASYMPTOTIC VALUES OF SLOWLY GROWING ALGEBROID FUNCTIONS

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1. Let $f(z)$ be a k-valued algebroid function in $|z| < \infty$ and

$$(1) \qquad F(z,f) \equiv A_0(z)f^k + A_1(z)f^{k-1} + \cdots A_k(z) = 0$$

be its defining equation such that the coefficients $A_i(z)$ $(i = 0, 1, \cdots, k)$ are entire functions without any common zero and the left hand side is irreducible. We denote by $\mathfrak{X}$ the k-sheeted covering surface over $|z| < \infty$ generated by $f(z)$ and by $\mathfrak{X}(r)$ and $\Gamma(r)$ the part of $\mathfrak{X}$ over $|z| \leq r$ and the curves on $\mathfrak{X}$ over $|z| = r$, respectively. We use the standard notations of the Nevanlinna-Selberg theory [4]:

$$m(r,a) = \frac{1}{2k\pi}\int_{\Gamma(r)} \log^+\left|\frac{1}{f(re^{i\theta}) - a}\right| d\theta, \quad m(r,f) = \frac{1}{2k\pi}\int_{\Gamma(r)} \log^+|f(re^{i\theta})|\, d\theta$$

$$N(r,a) = \frac{1}{k}\int_0^r \frac{n(t,a) - n(0,a)}{t} + \frac{n(0,a)}{k}\log r, \quad N(r,\infty) = N(r,f)$$

$$T(r,f) = m(r,f) + N(r,f), \qquad \delta(a,f) = 1 - \overline{\lim_{r\to\infty}}\frac{N(r,a)}{T(r,f)},$$

where $n(r,a)$ is the number of zeros of $f(z) - a$ on $\mathfrak{X}(r)$ and $n(r,\infty) = n(r,f)$.

From now on, we consider the functions with the slow growth:

$$(2) \qquad T(r,f) = O[(\log r)^2].$$

For such functions both of the number of deficient values and that of asymptotic values are at most k (Valiron [7], [9] and Tumura [5]). Especially, when $k = 1$ i.e. the function is single-valued and meromorphic, it can prossess no deficient value without that value being an asymptotic value (Valiron [9] and Anderson-Clunie [1]).

For an algebroid function $f(z)$, a value α is an asymptotic value, if there exists a path $L_{\mathfrak{X}}$ on $\mathfrak{X}$ stretching to the point at infinity such that $f(z)$

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