Hisasi Morikawa
Nagoya Math. J.
Vol. 41 (1971), 101–106

A NOTE ON HOLOMORPHIC VECTOR BUNDLES OVER COMPLEX TORI

HISASI MORIKAWA

1. Let $\omega : \boldsymbol{Z}^{2r} \to \boldsymbol{C}^{r}$ be an isomorphism of the free additive group of rank $2r$ into the complex vector n-space such that the quotient group $\boldsymbol{T} = \boldsymbol{C}^{r}/\omega(\boldsymbol{Z}^{2r})$ is compact, i.e., $\boldsymbol{T}_{\omega}$ is a complex torus.

We mean by a matric multiplier of rank n with respect to ω a family of complex holomorphic $n \times n$-matrix functions $\{\mu_{\alpha}(z)\}_{\alpha \in \boldsymbol{Z}^{2r}}$ on $\boldsymbol{C}^{r}$ such that

1) $\qquad \det \mu_{\alpha}(z) \neq 0 \quad (z \in \boldsymbol{C}^{r}),$

2) $\qquad \mu_{\alpha}(z)\mu_{\beta}(z + \omega(\alpha)) = \mu_{\alpha+\beta}(z), \quad (\alpha, \beta \in \boldsymbol{Z}^{2r}).$

By virtue of the conditions 1) and 2) we may define an action of $\boldsymbol{Z}^{2r}$ on the product $\boldsymbol{C}^{r} \times \boldsymbol{C}^{n}$ as follows:

$$(z, u) \to (z + \omega(\alpha),\ v\mu_{\alpha}(z)), \quad (\alpha \in \boldsymbol{Z}^{2r}).$$

The quotient $\boldsymbol{V}_{\mu}$ of $\boldsymbol{C}^{r} \times \boldsymbol{C}^{n}$ by this action of $\boldsymbol{Z}^{2r}$ is a holomorphic vector n-bundle over the complex torus $\boldsymbol{T}_{\omega}$, and conversely every holomorphic vector bundle over $\boldsymbol{T}_{\omega}$ is constructed by this method with a matric multiplier, since holomorphic vector bundles over a vector space are always trivial.[1)]

2. We shall recall the definition of *finite Heiseuberg groups* and their canonical representations.

Let G be an additive group of order n and of exponent d, and ζ be a primitive d-th root of unity. Let $\hat{G}$ be the dual group of G defined by a pairing $(\hat{a}, a) \to \langle \hat{a}, a \rangle$ of $\hat{G} \times G$ into the multiplicative group $\{1, \zeta, \cdots, \zeta^{d-1}\}$. We mean by the finite Heisenberg group $H(G)$ associated with G the group consisting of triples $\{(\hat{a}, a, \zeta^{l}) \mid \hat{a} \in \hat{G},\ a \in G,\ 0 \leq l \leq d-1\}$ with the composition law

$$(\hat{a}, a, \zeta^{l})(\hat{b}, b, \zeta^{h}) = (\hat{a} + \hat{b},\ a + b,\ \langle \hat{a}, b \rangle \zeta^{l+h}).$$

Received March 17, 1970.