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SOME GROUPS WHOSE S_3-SUBGROUPS HAVE MAXIMAL CLASS

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1. Introduction

In this paper, we investigate several classes of groups, among which the most general is defined as follows:

DEFINITION 1.1. A finite group G is a *SR-group* if it contains a subgroup P_1 of order 3 satisfying:

(a) A/S_3-subgroup P_2 of $N_G(P_1)$ is elementary of order 9;

(b) $N_G(P_2)/P_2$ acts semi-regularly by conjugation on the conjugates of P_1 contained in P_2.

To emphasize the role of P_1, we sometimes say G is a SR-group with respect to P_1.

The main result of this paper is

THEOREM 1.2. *If G is a SR-group, then $0^3(G)$ is a proper subgroup of G.*

It is clear that the definition of SR-groups can be easily generalized to primes other than 3, but the conclusion of Theorem 1.2 does not carry over to these primes.

The class of SR-groups contains several interesting subclasses, e.g., let X be a finite group, P_1 a subgroup of $\mathrm{Aut}(X)$ such that $|P_1| = |C_X(P_1)| = 3$. Then the semidirect product $G = P_1X$ is a SR-group. If $X = PSL(3, q)$, where q is congruent to 1 mod 3 but not mod 9, let α be the automorphism of X induced by the matrix $\begin{pmatrix} 0 & \lambda & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, where λ is a primitive cube root of unity in the field with q elements. If P_1 is the cyclic group generated by α, the semidirect product $G = P_1X$ is a SR-group.

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