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REMARKS ON THE ANGULAR DERIVATIVE*)

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Introduction. Suppose that Ω is a simply connected domain in the w-plane, $w = u + iv$, and that w_∞ is an accessible boundary point of Ω located at $w = \infty$. Suppose $w = W(z) = U(z) + iV(z)$ maps the strip $\sum = \{z = x + iy\colon\ -\infty < x < \infty,\ 0 < y < \pi\}$ conformally onto Ω such that $\lim_{x\to+\infty} W\left(x + i\frac{\pi}{2}\right) = w_\infty$. If in any sub-strip $\{z = x + iy\colon -\infty < x < \infty,$ $\delta \leqq y \leqq \pi - \delta\}$, $0 < \delta < \frac{\pi}{2}$,

$$\lim_{x\to+\infty} [W(z) - z] = \kappa \text{ exists and is finite,} \tag{1}$$

then $W(z)$ is said to have an angular derivative at $z = +\infty$.[1] The problem of finding geometrical conditions on Ω which ensure the existence of the angular derivative has received considerable attention ever since Carathéodory introduced this notion in the study of the boundary behavior of conformal maps in 1929 (cf. [5], Chapter III, [4], Chapter VI, in particular pp. 204–217, and [6], Theorem 6). In this note we present another such criterion, which for a wide class of domains yields a sharper sufficient condition than the earlier results. The basis for this criterion is the following more special result.

Suppose $\{u_n\}$, $\{v_n\}$, $\{v'_n\}$ are sequences of real numbers such that

$$u_{n+1} - u_n \geqq d > 0,\ \lim_{n\to\infty} v_n = 0,\ \lim_{n\to\infty} v'_n = \pi \tag{2}$$

and let S denote the interior of the union of the rectangles

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1) If Ω is mapped conformally onto a domain D such that w_∞ corresponds to a finite boundary point of D and $\sum$ onto the unit disk $\{|\zeta| < 1\}$ such that $z = +\infty$ corresponds to $\zeta = 1$, then the conformal mapping of the disk onto D has a non-vanishing finite derivative at $\zeta = 1$ for approach in a Stolz angle.