S. Ullom
Nagoya Math. J.
Vol. 39 (1970), 141–148

INTEGRAL NORMAL BASES IN GALOIS EXTENSIONS OF LOCAL FIELDS

S. ULLOM

Introduction

Throughout this paper F denotes a field complete with respect to a discrete valuation, k_F the residue field of F, K/F a finite Galois extension with Galois group $G = G(K/F)$†. The ring of integers O_K of K contains the (unique) prime ideal $\mathfrak{P}$; the collection of ideals $\mathfrak{P}^n$ for all integers n are ambiguous ideals i.e. G-modules. E. Noether [3] showed K/F tamely ramified implies O_K has an O_F-normal basis, i.e. is isomorphic as an O_FG-module to O_FG itself, O_FG the group ring of G over the ring O_F.

Define subgroups of G

$$G_{i^*} = \{\sigma \in G \mid \forall \alpha \in O_K,\ \sigma\alpha - \alpha \in \mathfrak{P}^{i+1}\},\ i \geqslant 0$$

and

$$G_i^* = \{\sigma \in G \mid \forall \alpha \in K^\times,\ \sigma\alpha/\alpha \in 1 + \mathfrak{P}^i\},\ i \geqslant 1.$$

Then $G_{i^*} \supset G_{i+1}^* \supset G_{i+1^*}$, $i \geqslant 0$, with $G_{i+1}^* = G_{i+1^*}$ written G_{i+1} if the residue field extension k_K/k_F is separable [2, p. 35]. We show (Theorem 3) that an ambiguous ideal $\mathfrak{A}$ of K has an O_F-normal basis iff the trace

$$S_{K/K_1}\mathfrak{A} = \mathfrak{A} \cap K_1,$$

where K_1 is the fixed field of the subgroup G_1^*. This result is obtained from the Galois module structure of $\mathfrak{A} \otimes_{O_F} F$ (resp. $\mathfrak{A} \otimes_{O_F} k_F$) where K/F is tamely ramified (resp. totally and wildly ramified).

Received November 11, 1968.
Revised June 6, 1969.

† Elements of Galois groups act on the left.

This research is supported by National Science Foundation Postdoctoral Fellowship Number 48037.