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ON PRINCIPAL FUNCTION PROBLEM

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To Professor Katsuji Ono on his sixtieth birthday

Sario's theory of principal functions fully discussed in his research monograph [3] with Rodin stems from the principal function problem which is to find a harmonic function p on an open Riemann surface R imitating the ideal boundary behavior of the given harmonic function s in a neighborhood A of the ideal boundary δ of R. The mode of imitation of p to s is described by a linear operator L of functions on ∂A into harmonic functions on A: p imitates the behavior of s at the ideal boundary if $L((p-s)|\partial A) = p|A - s$. Sario [4] considered normal operators L in his terminology as imitative operators and gave a complete solution to the principal function problem with respect to his class of operators.

Recently Yamaguchi [5] introduced a new class of imitative operators, the class of regular operators in his terminology, and also gave an existence theorem in the principal function problem. Sario's class of imitative operators is neither contained in nor does contain that of Yamaguchi.

Therefore it is desirable to introduce a wider class of imitative operators which contains those considered by Sario and Yamaguchi and also to obtain the complete solution to the principal function problem with respect to this new class of operators, which is the object of the present note. In this context refer also to Nakai [2].

1. Throughout this note we will denote by R an open Riemann surface. However the whole argument in the sequel can be applied without any change to the case where R is replaced by a noncompact Riemannian manifold of an arbitrary dimension whose base manifold is orientable, connected, separable, and of class C^2 and whose metric tensor possesses Hölder continuous first order derivatives.

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