H. Kunita
Nagoya Math. J.
Vol. 38 (1970), 41–52

STOCHASTIC INTEGRALS BASED ON MARTINGALES TAKING VALUES IN HILBERT SPACE

HIROSHI KUNITA

To Professor Katuzi Ono on the occasion of his 60th birthday

Let H be a separable Hilbert space with inner product $(\ ,\ )$ and norm $\|\ \|$. We denote by K the set of all linear operators on H. Let $(\Omega, \mathfrak{F}, P)$ be a probability space and suppose we are given a family of σ-fields $\mathfrak{F}_t$, $t \geq 0$ such that $\mathfrak{F}_s \subseteqq \mathfrak{F}_t \subseteqq \mathfrak{F}$ for $0 \leq s \leq t$ and $\bigcap_{\varepsilon>0} \mathfrak{F}_{t+\varepsilon} = \mathfrak{F}_t$. We assume further that each $\mathfrak{F}_t$ is complete relative to the probability measure P. A mapping $X_t(\omega)$; $[0, \infty) \times \Omega \to H$ is called an *H-valued stochastic process* or shortly *H-process* if (f, X_t) is a scalar valued (real or complex) stochastic process for all $f \in H$. In particular, if (f, X_t) is a martingale for every $f \in H$, X_t is called an *H-martingale*.

The purpose of this article is to define two types of stochastic integrals by H-martingale $\int_0^t (\Phi_1(s, \omega), dX_s(\omega))$ and $\int_0^t \Phi_2(s, \omega) dX_s(\omega)$ and to establish a formula concerning these stochastic integrals. Here $\Phi_i(s, \omega)$, $i = 1, 2$ is H- or K-process, respectively, with suitable additional conditions. Similar problem concerning Hilbert space valued Brownian motion has been discussed by Daletskii [1].

1. **Preliminaries.** Let X be an H-random variable. Then $\|X(\omega)\|$ is clearly an $\mathfrak{F}$-measurable real random variable. We suppose $E\|X\| < \infty$. For a given sub σ-field $\mathfrak{G}$ of $\mathfrak{F}$, we define the *conditional expectation* of X relative to $\mathfrak{G}$, denoted by $E(X|\mathfrak{G})$, in the following manner; $E(X|\mathfrak{G})$ is an H-random variable such that $(f, E(X|\mathfrak{G}))$ is $\mathfrak{G}$-measurable and $(f, E(X|\mathfrak{G})) = E((f, X)|\mathfrak{G})$ holds for every $f \in H$. Such $E(X|\mathfrak{G})$ is unique up to measure 0. Then an H-process X_t such that $E\|X_t\| < \infty$, $\forall t \geq 0$, is an H-martingale if and only if $E(X_t|\mathfrak{F}_s) = X_s$ holds for every $t \geq s$.

Received March 31, 1969