

**ON SOME DOUBLY TRANSITIVE PERMUTATION  
GROUPS OF DEGREE  $N$   
AND ORDER  $6n(n - 1)$**

SHIRO IWASAKI AND HIROSHI KIMURA<sup>1)</sup>

*Dedicated to Professor K. Ono on his 60th birthday*

The purpose of this paper is to prove the following result.

**THEOREM.** *Let  $\Omega$  be the set of symbols  $1, 2, \dots, n$ . Let  $\mathcal{G}$  be a doubly transitive group on  $\Omega$  of order  $6n(n - 1)$  not containing a regular normal subgroup and let  $\mathfrak{R}$  be the stabilizer of the set of symbols 1 and 2. Assume that  $\mathfrak{R}$  is cyclic and independent, i.e.,  $\mathfrak{R} \cap G^{-1}\mathfrak{R}G = 1$  or  $\mathfrak{R}$  for every element  $G$  of  $\mathcal{G}$ . Then  $\mathcal{G}$  is isomorphic to either  $PGL(2, 7)$  or  $PSL(2, 13)$ .*

We use the standard notation;

$C_{\mathfrak{X}}(\mathfrak{X})$ : the centralizer of a subset  $\mathfrak{X}$  in a group  $\mathfrak{X}$

$N_{\mathfrak{X}}(\mathfrak{X})$ : the normalizer of  $\mathfrak{X}$  in  $\mathfrak{X}$

$\langle \dots \rangle$ : the subgroup generated by  $\dots$

$|\mathfrak{X}|$ : the number of elements in  $\mathfrak{X}$

$[\mathfrak{X} : \mathfrak{Y}]$ : the index of a subgroup  $\mathfrak{Y}$  in  $\mathfrak{X}$

$\mathfrak{X}^G$ :  $G^{-1}\mathfrak{X}G$  where  $G \in \mathfrak{X}$ .

*Proof of Theorem*

1. Let  $\mathfrak{S}$  be the stabilizer of the symbol 1.  $\mathfrak{R}$  is of order 6 and it is generated by a permutation  $K$  whose cyclic structure has the form  $(1)(2) \dots$ . Since  $\mathcal{G}$  is doubly transitive on  $\Omega$ , it contains an involution  $I$  with the cyclic structure  $(1, 2) \dots$  which is conjugate to  $K^3$ . Then we have the following decomposition of  $\mathcal{G}$ ;

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