Shizuo Endo
Nagoya Math. J.
Vol. 36 (1969), 121–141

ON RAMIFICATION THEORY IN PROJECTIVE ORDERS

SHIZUO ENDO

The ramification theory in commutative rings, as a generalization of the classical one in maximal orders over a Dedekind domain, was established in [1], [13], [15] and etc.. For non-commutative algebras this was also studied in [3], [4], [9], [18] and etc.. However, the different theorem, which is a central part of ramification theory, has not been given in those, except for some special cases (cf. [12], [18]). The main object of this paper is to give the discriminant theorem and the different theorem for projective orders in a (non-commutative) separable algebra, in the most general form.

Let R be a Dedekind domain and K be the quotient field of R. Recently it was proved in [9] that, if R has the perfect residue class fields, then the Noetherian different of a maximal R-order in a central simple K-algebra is a square-free ideal of R. Another object of this paper is, more generally without assuming that R has the perfect residue class fields, to determine completely the structure of the Dedekind different and the Noetherian different for a hereditary R-order in a central simple K-algebra.

In § 2 we discuss some basic properties of the differents of algebras and give criteria on the separability of projective orders. In § 3 we first give the Noetherian different theorem for algebras. Further, using these results, we prove, under some restrictive assumptions, the discriminant theorem and the Dedekind different theorem for projective orders, each of which is a generalization of the classical one for maximal orders over a Dedekind domain.

In § 4 we restrict our attention to hereditary orders over a Dedekind domain. Let R be a Dedekind domain with quotient field K and $\sum$ a central simple K-algebra. We show that, for any hereditary R-order Λ in

Received December 14, 1968.
This work was supported in part by the Matsunaga Science Foundation.